

Quantum entanglement for two qubits in a nonstationary cavity

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The quantum entanglement and the probability of the dynamical Lamb effect for two qubits caused by non-adiabatic fast change of the boundary conditions are studied. The conditional concurrence of the qubits for each fixed number of created photons in a nonstationary cavity is obtained as a measure of the dynamical quantum entanglement due to the dynamical Lamb effect. We discuss the physical realization of the dynamical Lamb effect, based on superconducting qubits.

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I. INTRODUCTION

The new direction in the quantum electrodynamics (QED), related to the optical properties of the cavity with the nonstationary boundary conditions, initiated studies of several interesting physical phenomena such as a dynamical Casimir effect (DCE) [1–4]. Casimir predicted that two mirrors, which are perfectly conducting metal plates, held parallel to each other in vacuum, will experience an attractive force [5]. Essentially, the mirrors reduce the density of the electromagnetic modes between them. The vacuum radiation pressure between the plates is then less than the pressure outside, that generates the attractive force. This effect of quantum electrodynamics is known as the static Casimir effect. If the mirrors move sufficiently fast, a mismatch between vacuum modes at different times appears. It has been predicted that this results in creation of real photons out of vacuum fluctuations called the dynamical Casimir effect (DCE) [1]. The control of the Lamb shift in a nonstationary cavity due to adiabatical change of the cavity modes was also considered [6]. The new effect of cavity quantum electrodynamics, the dynamical Lamb effect (DLE), which is photonless, parametric excitation of an atom, embedded in a nonstationary cavity, by shaking its photonic coat due to nonadiabatic change of the boundary conditions for virtual photons, contributing to the electronic self-energy, was predicted [7].

The fast changes of the boundary conditions can be achieved in superconducting circuits. The superconducting qubits play the role of atoms in an optical cavity, and the superconducting line plays the role of an optical cavity [8–13]. The fast changes of the boundary conditions can be achieved by using superconducting quantum interference device (SQUID) with the fast changes of the external magnetic field. It was shown that a coplanar waveguide (CPW) terminated by a SQUID is a very promising system for experimental observation of the dynamical Casimir effect for the GHz frequency range instead of optical region for regular atoms in an optical cavity [14, 15]. The observation of the DCE requires to go sufficiently beyond the regime of adiabatic change of the boundary conditions of the cavity. Non-adiabatic change of these boundary conditions corresponds to the requirement that the characteristic time of the change of the boundary conditions must be smaller than the period of oscillations of the electromagnetic field, formed due to the DCE. This nonadiabatic change of the boundary conditions requires that the frequency of the photon, excited due to the DCE, must be smaller than the frequency of the change of the boundary conditions. The high quality factor of the superconducting cavity and the possibilities for easy control (using SQUID) of the boundary conditions with the frequencies of the order of magnitude of the eigenfrequencies of the superconducting cavity allow to observe DCE and DLE for the superconducting qubits.

In this paper, we consider the physical realization to observe the quantum entanglement and the DLE for the artificial atoms, formed by superconducting qubits, constructed in a superconducting circuit based on Josephson junctions. We analyze superconducting qubits, coupled to a CPW terminated by a SQUID. Changing the magnetic flux through the SQUID parametrically modulates the boundary conditions of CPW. Ultrafast change of CPW boundary conditions results in the excitation of the superconducting qubit which is identical to the dynamical Lamb effect for an ordinary atom in a cavity with the nonstationary changing parameters. We focus on the physical realization of the DLE [7] resulting in excitation of two qubits. While the influence of the modulation of the constant of the qubit-photon interaction on excitation of the qubit was studied in Ref. 16, below we propose excitation of the qubits due to the change of the boundary conditions of the cavity. We analyze the quantum entanglement and obtain the probability of the DLE for two qubits, coupled to an optical cavity.

The paper is organized in the following way. In Sec. II, we study the dynamical Lamb effect for two qubits, coupled

to an optical cavity with changing boundary conditions, and derive the probability of the DLE. In Sec. III, we analyze the quantum entanglement for two qubits caused by nonadiabatic fast change of the boundary conditions, and obtain the conditional concurrence for two qubits at fixed number of created photons in a nonstationary cavity. Finally, the discussion of the results and conclusions follow in Sec. IV.

II. THE DYNAMICAL LAMB EFFECT FOR TWO QUBITS, COUPLED TO AN OPTICAL CAVITY

We consider two qubits, coupled to an optical cavity with changing boundary conditions. The Hamiltonian $\hat{\mathcal{H}}$ of two qubits in a nonstationary cavity, assuming $\hbar = 1$, is given by (the Hamiltonian of one qubit in a nonstationary cavity was presented in Ref. 7)

$$\hat{\mathcal{H}} = \hat{H} + i \frac{\dot{\omega}(t)}{4\omega(t)} (a^2 - a^{\dagger 2}) , \quad (1)$$

where \hat{H} is the Hamiltonian of a stationary cavity, defined as

$$\hat{H} = \hat{H}_0 + \hat{V}_{total} , \quad (2)$$

where \hat{H}_0 is the Hamiltonian of two qubits without qubit-photon interaction, which is given by

$$\hat{H}_0 = E_0 \sum_{j=1,2} \frac{1 + \hat{\sigma}_{3j}}{2} + \omega a^{\dagger} a , \quad (3)$$

\hat{V}_{total} is the total Hamiltonian of the qubit-photon interaction given by

$$\hat{V}_{total} = \lambda \sum_{j=1,2} (\hat{\sigma}_j^+ + \hat{\sigma}_j^-) (a + a^{\dagger}) . \quad (4)$$

In Eqs. (3) and (4), $\hat{\sigma}^+ = (\hat{\sigma}_1 + i\hat{\sigma}_2)/2$, $\hat{\sigma}^- = (\hat{\sigma}_1 - i\hat{\sigma}_2)/2$, assuming $\hat{\sigma}_1$, $\hat{\sigma}_2$, and $\hat{\sigma}_3$ are the Pauli matrices, a^{\dagger} and a are creation and annihilation operators for cavity photons, correspondingly, E_0 is the qubit transition frequency, λ is the strength of the artificial qubit-photon coupling. The second term in the r.h.s. of Eq. (1) was obtained in Ref. 17.

It follows from Eq. (4) that

$$\hat{V}_{total} = \hat{V}_{RWA} + \hat{V} , \quad (5)$$

where \hat{V}_{RWA} is the qubit-photon interaction in the rotating wave approximation (RWA)

$$\hat{V}_{RWA} = \lambda \sum_{j=1,2} (\hat{\sigma}_j^+ a + \hat{\sigma}_j^- a^{\dagger}) , \quad (6)$$

which does not change the number of the excitations in the system, and \hat{V} is the term beyond the RWA, which can increase or decrease the number of the excitations in the system for two or four excitations, including the qubit and photon excitations:

$$\hat{V} = \lambda \sum_{j=1,2} (\hat{\sigma}_j^+ a^{\dagger} + \hat{\sigma}_j^- a) . \quad (7)$$

Since the DLE is related to the parametric excitation of the qubit, only \hat{V} influences the DLE.

The second term in Eq. (1) describes the dynamical Casimir effect, parametric excitation of photons due to non-adiabatical change of the cavity photon frequency. There are three channels of qubit excitations in a nonstationary optical cavity due to non-adiabatical change of the cavity eigenfrequency. The first and second channels correspond to excitation of a qubit by real and virtual Casimir photons, correspondingly. This paper is focused on the third channel, which corresponds to a new effect of nonstationary quantum electrodynamics, the dynamical Lamb effect, which is parametric excitation of an atom due to modulation of its Lamb shift or, in other words, due to shaking of its photonic coat.

Using Eqs. (3) and (4), one gets the eigenstate vectors of the Hamiltonian \hat{H} in the framework of the first order perturbation theory [18]:

$$\begin{aligned}
|n; 00\rangle_{\lambda\omega} &= |n; 00\rangle + \frac{\lambda\sqrt{n}(|n-1; 10\rangle + |n-1; 01\rangle)}{\omega - E_0} - \frac{\lambda\sqrt{n+1}(|n+1; 10\rangle + |n+1; 01\rangle)}{\omega + E_0}, \\
|n; 10\rangle_{\lambda\omega} &= |n; 10\rangle + \frac{\lambda\sqrt{n}|n-1; 00\rangle - \lambda\sqrt{n+1}|n+1; 11\rangle}{\omega + E_0} + \frac{\lambda\sqrt{n}|n-1; 11\rangle - \lambda\sqrt{n+1}|n+1; 00\rangle}{\omega - E_0}, \\
|n; 01\rangle_{\lambda\omega} &= |n; 01\rangle + \frac{\lambda\sqrt{n}|n-1; 00\rangle - \lambda\sqrt{n+1}|n+1; 11\rangle}{\omega + E_0} + \frac{\lambda\sqrt{n}|n-1; 11\rangle - \lambda\sqrt{n+1}|n+1; 00\rangle}{\omega - E_0}, \\
|n; 11\rangle_{\lambda\omega} &= |n; 11\rangle + \frac{\lambda\sqrt{n}(|n-1; 10\rangle + |n-1; 01\rangle)}{\omega + E_0} - \frac{\lambda\sqrt{n+1}(|n+1; 10\rangle + |n+1; 01\rangle)}{\omega - E_0},
\end{aligned} \tag{8}$$

where n is the number of photons.

Using Eqs. (3) and (4), we get the eigenvalues of the Hamiltonian \hat{H} in the framework of the second order perturbation theory [18]:

$$\begin{aligned}
E_{n;00}^{(\lambda)} &= \left(\omega + \frac{4\lambda^2 E_0}{\omega^2 - E_0^2} \right) n - \frac{2\lambda^2}{\omega + E_0}, \\
E_{n;10}^{(\lambda)} &= E_{n;01}^{(\lambda)} = n\omega + E_0 + \frac{2\lambda^2 \omega}{\omega^2 - E_0^2}, \\
E_{n;11}^{(\lambda)} &= \left(\omega - \frac{4\lambda^2 E_0}{\omega^2 - E_0^2} \right) n + 2E_0 - \frac{2\lambda^2}{\omega - E_0}.
\end{aligned} \tag{9}$$

The second order with respect to qubit-photon coupling λ terms in Eqs. (9) are not proportional to n and can be interpreted as Lamb shifts of qubit levels, since the number of photons is not changing due to the static Lamb shift, because the virtual photons are excited and absorbed. These terms, corresponding to the total Lamb shifts of two qubits in different states are given by the following expressions:

$$\begin{aligned}
E_{L,00}^{(\lambda)}(\omega) &= -\frac{2\lambda^2}{\omega + E_0}, \\
E_{L,10}^{(\lambda)}(\omega) &= E_{L,01}^{(\lambda)} = \frac{2\lambda^2 \omega}{\omega^2 - E_0^2}, \\
E_{L,11}^{(\lambda)}(\omega) &= -\frac{2\lambda^2}{\omega - E_0}.
\end{aligned} \tag{10}$$

In the framework of the model, the total Lamb shift is the sum of the Lamb shifts of the individual qubits. The static Lamb shift is obtained by the second order perturbation theory, since it involves the creation and absorption of the virtual photons by the qubit described by the Feynman's diagram for the self-energy of the second order. The latter contains two vertexes for the qubit-photon coupling [19].

If the cavity resonance frequency is changed suddenly from ω_1 to ω_2 , the cavity photons can be created, and the qubits can get an excitation due to change of the parameters of the cavity. The amplitudes $A_{n;10}^{(L)}$ and $A_{n;01}^{(L)}$ for excitation of one qubit and creation of n cavity photons are given by

$$A_{n;10}^{(L)} =_{\lambda\omega_2} \langle n; 10 | 0; 00 \rangle_{\lambda\omega_1}, \quad A_{n;01}^{(\lambda)} =_{\lambda\omega_2} \langle n; 01 | 0; 00 \rangle_{\lambda\omega_1}. \tag{11}$$

The amplitude $A_{n;11}^{(L)}$ for excitation of two qubits and creation of n cavity photons is given by

$$A_{n;11}^{(L)} =_{\lambda\omega_2} \langle n; 11 | 0; 00 \rangle_{\lambda\omega_1}. \tag{12}$$

The probabilities of DLE $w_{n;10}^{(L)}$ and $w_{n;01}^{(L)}$ of excitation of one qubit are given by

$$\begin{aligned}
w_{n;10}^{(L)} &= \sum_{n=0}^{\infty} \left| A_{n;10}^{(L)} \right|^2 = \left| A_{1;10}^{(L)} \right|^2 = |_{\lambda\omega_2} \langle 1; 10 | 0; 00 \rangle_{\lambda\omega_1}|^2 = \lambda^2 \left(\frac{1}{\omega_2 + E_0} - \frac{1}{\omega_1 + E_0} \right)^2 \\
&= \lambda^2 \left(-\frac{E_{L;00}^{(\lambda)}(\omega_2)}{2\lambda^2} + \frac{E_{L;00}^{(\lambda)}(\omega_1)}{2\lambda^2} \right)^2 = \left(\frac{\delta E_{L;00}^{(\lambda)}}{2\lambda} \right)^2,
\end{aligned}$$

$$\begin{aligned}
w_{n;01}^{(L)} &= \sum_{n=0}^{\infty} \left| A_{n;01}^{(L)} \right|^2 = \left| A_{1;01}^{(L)} \right|^2 = \left| {}_{\lambda\omega_2} \langle 1; 01 | 0; 00 \rangle_{\lambda\omega_1} \right|^2 = \lambda^2 \left(\frac{1}{\omega_2 + E_0} - \frac{1}{\omega_1 + E_0} \right)^2 \\
&= \lambda^2 \left(-\frac{E_{L;00}^{(\lambda)}(\omega_2)}{2\lambda^2} + \frac{E_{L;00}^{(\lambda)}(\omega_1)}{2\lambda^2} \right)^2 = \left(\frac{\delta E_{L;00}^{(\lambda)}}{2\lambda} \right)^2,
\end{aligned} \tag{13}$$

where

$$A_{1;10}^{(L)} = A_{1;01}^{(L)} = \lambda \left(\frac{1}{\omega_2 + E_0} - \frac{1}{\omega_1 + E_0} \right). \tag{14}$$

Note that excitation of one qubit is possible only with creation of one cavity photon.

Let us estimate the probability of excitation of a single qubit due to the DLE by using the physical realization with the superconducting qubits, coupled to CPW. Following Ref. 20 we use the parameters: $\omega_1 = 2\pi \times 5$ GHz, $E_0 = 2\pi \times 3.721$ GHz, $\lambda = 0.04\omega_1 = 2\pi \times 0.2$ GHz. According to Ref. 14, assuming the relative change of the Josephson energy and the length of the transmission line, corresponding to the change of the boundary conditions of the cavity, is 0.25, we use $\omega_2 = \omega_1 - 0.25\omega_1 = 2\pi \times 3.75$ GHz. Substituting listed above parameters into Eq. (13), one obtains $w_{n;10}^{(L)} = w_{n;01}^{(L)} = 1.472 \times 10^{-5}$.

The contribution of DLE to the probability $w_{n;11}^{(L)}$ of excitation of two qubits is given by

$$w_{n;11}^{(L)} = \sum_{n=0}^{\infty} \left| A_{n;11}^{(L)} \right|^2 = \left| A_{0;11}^{(L)} \right|^2 + \left| A_{2;11}^{(L)} \right|^2, \tag{15}$$

where $A_{0;11}^{(L)}$ is the amplitude of excitation of two qubits without creation of cavity photons given by

$$A_{0;11}^{(L)} = {}_{\lambda\omega_2} \langle 0; 11 | 0; 00 \rangle_{\lambda\omega_1} = \frac{2\lambda^2}{(\omega_1 + E_0)(\omega_2 - E_0)}, \tag{16}$$

and $A_{2;11}^{(L)}$ is the amplitude of excitation of two qubits with creation of two cavity photons given by

$$A_{2;11}^{(L)} = {}_{\lambda\omega_2} \langle 2; 11 | 0; 00 \rangle_{\lambda\omega_1} = -\frac{2\sqrt{2}\lambda^2}{(\omega_1 + E_0)(\omega_2 + E_0)}. \tag{17}$$

According to Eq. (15), the process with excitation of two qubits is possible only without creation of photons or with creation of two photons.

Substituting Eqs. (16) and (17) into Eq. (15), one gets

$$w_{n;11}^{(L)} = \frac{4\lambda^4}{(\omega_1 + E_0)^2} \left(\frac{1}{(\omega_2 - E_0)^2} + \frac{2}{(\omega_2 + E_0)^2} \right) = \left(E_{L;00}^{(\lambda)}(\omega_1) \right)^2 \left(\left(\frac{E_{L;11}^{(\lambda)}(\omega_2)}{2\lambda^2} \right)^2 + 2 \left(\frac{E_{L;00}^{(\lambda)}(\omega_2)}{2\lambda^2} \right)^2 \right). \tag{18}$$

Let us mention that the probabilities of excitation of qubits are expressed in terms of the Lamb shifts.

We estimate the probability of excitation of two qubits due to the DLE by using the physical realization with the superconducting qubits, coupled to CPW. Applying listed above parameters in Eq. (18), one obtains $w_{n;11}^{(L)} = 0.1$. According to Eq. (18), $w_{n;11}^{(L)}$ is enhanced essentially, when E_0 is near the resonance with ω_2 . Let us mention that since the probabilities of excitations of qubits due to the DLE $w_{n;10}^{(L)}$, $w_{n;01}^{(L)}$, and $w_{n;11}^{(L)}$ are essentially less than one, the application of the perturbation theory used above for the system under consideration is valid.

III. QUANTUM ENTANGLEMENT FOR TWO QUBITS DUE TO THE CHANGE OF THE BOUNDARY CONDITIONS IN THE CAVITY

Note that the probability of creation of two photons due to the dynamical Lamb effect in Eq. (18) is not given by the product of the probabilities of creation of a single photon presented in Ref. 7. Therefore, the quantum entanglement for two qubits occurs due to the dynamical Lamb effect.

It is well-known that the possible measure of quantum entanglement of two qubits is the concurrence C [21–23], defined for the two-qubit state $|\Phi\rangle$

$$|\Phi\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle \quad (19)$$

as

$$C(\Phi) = 2|ad - bc| \quad (20)$$

We define the conditional concurrence for each fixed number of created photons. For the case of absence of photons $|0\rangle_{ph}$, we have $a = A_{0;00}^{(L)}$, $d = A_{0;11}^{(L)}$, $b = 0$, $c = 0$. For the case of creation one photon $|1\rangle_{ph}$, we have $b = A_{1;01}^{(L)}$, $c = A_{1;10}^{(L)}$, $a = 0$, $d = 0$. In this case, the conditional concurrence for one photon $C_{|1\rangle}$ is defined as

$$C_{|1\rangle} = 2|bc| = 2\left|A_{1;01}^{(L)}A_{1;10}^{(L)}\right| \quad (21)$$

Substituting Eq. (14) into Eq. (21), we obtain

$$C_{|1\rangle} = 2\lambda^2 \left(\frac{1}{\omega_2 + E_0} - \frac{1}{\omega_1 + E_0} \right)^2 = 2w_{n;10}^{(L)} = 2w_{n;01}^{(L)} = 2 \left(\frac{\delta E_{L;00}^{(\lambda)}}{2\lambda} \right)^2 \quad (22)$$

Let us mention that the concurrence is expressed in terms of the variation of the Lamb shift.

One can estimate the conditional concurrence for one created photon due to the DLE by using the physical realization with the superconducting qubits, coupled to CPW. Substituting used above parameters into Eq. (22), one obtains $C_{|1\rangle} = 2.945 \times 10^{-5}$.

For the case of creation two photons $|2\rangle_{ph}$, we have $a = A_{2;00}^{(L)}$, $d = A_{2;11}^{(L)}$, $b = 0$, $c = 0$. In this case, the conditional concurrence for two photons $C_{|2\rangle}$ is defined as

$$C_{|2\rangle} = 2|ad| = 2\left|A_{2;00}^{(L)}A_{2;11}^{(L)}\right| \quad (23)$$

Substituting Eq. (17) and the amplitude of the creation of two photons without excitation of a qubit $A_{2;00}^{(L)}$, given by

$$A_{2;00}^{(L)} = {}_{\lambda\omega_2} \langle 2;00 | 0;00 \rangle_{\lambda\omega_1} = -\frac{2\sqrt{2}\lambda^2}{(\omega_1 + E_0)(\omega_2 - E_0)} \quad (24)$$

into Eq. (23), we obtain

$$C_{|2\rangle} = \frac{16\lambda^4}{(\omega_1 + E_0)^2 |\omega_2^2 - E_0^2|} \quad (25)$$

We estimate the conditional concurrence for two created photons due to the DLE by using the physical realization with the superconducting qubits, coupled to CPW. Applying previously used parameters in Eq. (25), one obtains $C_{|2\rangle} = 1.553 \times 10^{-3}$. According to Eq. (25), $C_{|2\rangle}$ is enhanced essentially, when E_0 is near the resonance with ω_2 . Note that the probability of excitation of two qubits and the corresponding conditional concurrence can be essentially enhanced by multiple or periodic excitation due to change of the boundary conditions. See, e.g., Ref. 20 for another physical realization, namely the dynamical Casimir effect for two cavities with variable coupling and entanglement of qubits, coupled to the cavities; another case is periodic excitation of coupling with cavity [16]. The periodic excitation of the cavity with coupled qubits and also the role of the damping of qubits and cavity will be considered in another study.

Let us mention, that if one of the qubits is in the excited state, while another one is in the ground state, and the qubits are in the state either $|10\rangle_{qub}$ or $|01\rangle_{qub}$, there is only one photon in the system. We conclude that if both qubits are either in the ground state $|00\rangle_{qub}$ or both qubits are in the excited state $|11\rangle_{qub}$, then the photon field in the system is entangled between the states with absence of photons $|0\rangle_{ph}$ and two photons $|2\rangle_{ph}$. The probabilities of each photon state in the cases with both qubits in the ground $|00\rangle_{qub}$ and excited $|11\rangle_{qub}$ states are different. Thus, if both qubits are in the ground state $|00\rangle_{qub}$, the probability of two created photons in the system is given by $\left|A_{2;00}^{(L)}\right|^2$. The amplitude $A_{2;00}^{(L)}$ is given by Eq. (24). If both qubits are in the excited state $|11\rangle_{qub}$, the probability of absence of created photons in the system is given by $\left|A_{0;11}^{(L)}\right|^2$, and the probability of two created photons in the system is given by $\left|A_{2;11}^{(L)}\right|^2$. The amplitudes $A_{0;11}^{(L)}$ and $A_{2;11}^{(L)}$ are given by Eqs. (16) and (17), correspondingly.

IV. DISCUSSION AND CONCLUSIONS

The superconducting circuits have the advantage of enabling the study of complex controllable quantum dynamics. This could lead to quantum simulations and on-chip studies of many-body physics. Two-qubit algorithms with a superconducting quantum processor were demonstrated in Ref. [24]. The entangled photon states with the different frequencies excited due to the DCE in a superconducting circuit can be applied for the quantum information. The transfer of signals associated with the photons with the different frequencies and/or different polarizabilities can be used as the basis for novel developments in the quantum cryptography. These signals can be created due to the DCE in a superconducting circuit. We consider the physical realization to observe the dynamical Lamb effect in the superconducting qubits which is the novel physical phenomenon, that can be observed experimentally. The numerous novel phenomena and applications will be discovered using superconducting circuits, and these effects will play an important role in future quantum technologies.

We evaluated the quantum entanglement and the probability of the DLE for two qubits, coupled to the optical cavity (or the superconducting line for superconducting realizations), caused by nonadiabatic fast change of the boundary conditions. We obtained the probability of the dynamical Lamb effect. According to Eq. (18), the probability of excitation of two qubits is not equal to the product of the probabilities of excitation of the first and second qubits. This is the difference between the dynamic Lamb effect in one and two qubits. As a measure of the dynamical quantum entanglement, the conditional concurrence for two qubits for each fixed number of created photons in a nonstationary cavity was derived and analyzed. We considered the quantum entanglement not caused by interaction between two qubits, but due to change of the boundary conditions of the cavity.

The system with two qubits is characterized by their influence on each other and their quantum entanglement with each other and created Casimir photons due to the change of the boundary conditions. This effect is expected to play an important role in quantum technologies. The dynamical Lamb effect and quantum entanglement due to the change of cavity boundary conditions offer a new possibility of control of qubits and open new directions for quantum technologies. These phenomena related to the DLE can be observed at the different frequencies regions either for atoms or for semiconductor quantum dots in the optical cavity or for the superconducting qubits, coupled to the superconducting line. The model Hamiltonian, which we use in the present article, qualitatively describes the DLE for all three physical realizations.

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